Stein has suggested that the Nunge-Gill procedure is weakest in the plug flow case for values of KH which are very different from unity. To illustrate the convergence of the effectiveness as successive terms, Table 2 gives values of ϵ_0 and ϵ_L for $KH = 4.5$, the maximum KH product of the cases investigated previously. It is clear from Table 2 that ϵ_0 and ϵ_L are the same and unchanging for $M = 5$ and that the converged value agrees with ϵ_{N-G} in Table 1. Hence, $KH = 4.5$ is not sufficiently different from unity to reflect any significant differences between the computational procedures.

As is almost always the case, it is necessary, when one applies the general theory to a particular physical problem, to examine the related numerical calculations carefully. For the laminar flow case we did this originally by checking the internal consistency and by generating a finite-difference solution for comparison. We have done it again by using an alternate numerical scheme for solving the same linear system of equations. Now we are even more confident that our results for the laminar flow case are accurate enough for the practical purpose of calculating effectiveness, local temperature distributions, and Nusselt numbers. However, it is clear that when one attacks a different physical problem, the numerical techniques employed will have to be tailored to fit the needs of that particular problem.

A different method of solving the countercurrent problem, which employs the Duhamel theorem, has been described briefly by Blanco and Gill (5). In this case the problem is reduced to solving a Fredholm equation of the first kind.

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A Method of Predicting Boiling Pressure Drop for Alkali Metals

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Recently, some alkali-metal boiling pressure-drop data were shown to agree with the constant-slip model boiling pressure-drop correlation of Stone and Damman (*1*). The model assumed in developing the correlation was similar to that of Thom (*2*), in that the slip ratio (mean gas velocity divided by mean liquid velocity) is assumed to be a function of the liquid-to-gas density ratio only. Thom indicates that his method follows the lines suggested by Martinelli and Nelson (*3*) for a horizontal tube, but it has been extended to include the vertical-flow case, whereas Lockhart and Martinelli (*4*) do not include gravitational head effects.

Some modifications of Thom's method (*2*) (based on high pressure, water boiling data) are required in order to correlate low pressure, water boiling data (*1*). The correlation was based on data of the authors (*1*), and of Dengler (*5*) for low pressure water boiling in vertical upflow heat exchangers and on some unpublished NASA sodium boiling data (liquid-to-vapor density ratios from about 800 to 6,000). After the development of this correlation, additional NASA sodium boiling data was obtained, and Peterson (*6*) presented the results of an extensive potassium boiling investigation. The agreement between these data and the correlation of reference *1* suggests that this correlation may be useful in predicting pressure drop for alkali-metal boilers.

ANALYSIS

Application of the conservation of energy, mass, and momentum yields the pressure drop as the sum of three terms: inertial, frictional, and gravitational. The results of Thom (*2*) for constant heat flux, friction factor, slip ratio, and physical properties are given below:

$$\Delta P_I = \frac{G^2}{144\rho_l g_c}\left\{\left[1 + x_E\left(\frac{1}{V}\,\frac{\rho_l}{\rho_g} - 1\right)\right][1 + x_E(V-1)] - 1\right\} \quad (1)$$

$$\Delta P_F = \frac{f_{TP}G^2}{144\rho_l g_c}\left\{\left[1 + x_E\left(\frac{1}{V}\,\frac{\rho_l}{\rho_g} - 1\right)\right][1 + x_E\,(V-1)] + 1\right\}\frac{L}{D} \quad (2)$$

$$\Delta P_G = \frac{\rho_l L}{144}\left(\frac{g}{g_c}\right)\left\{\frac{\frac{1}{V}\left(\frac{\rho_l}{\rho_g} - 1\right)}{\left(\frac{1}{V}\,\frac{\rho_l}{\rho_g} - 1\right)^2 x_E}\ln\left[1 + x_E\left(\frac{1}{V}\,\frac{\rho_l}{\rho_g} - 1\right)\right] + \frac{\frac{1}{V} - 1}{\frac{1}{V}\,\frac{\rho_l}{\rho_g} - 1}\right\} \quad (3)$$

The inertial pressure drop ΔP_I is a function of the mean density and velocity at the inlet and exit, and is independent of the local heat flux distribution within the boiler. Uniform heat flux was assumed in order to obtain the frictional and gravitational pressure drop equations (2) and (3). Equation (3) applies only to vertical upflow boilers; for the horizontal case $\Delta P_G = 0$, and for the vertical downflow case ΔP_G is the negative of Equation (3).

In order to use Equations (1), (2), and (3), one must know the parameter V. In developing the correlation of reference *1*, trial and error showed that the approximation $V = \sqrt{\rho_l/\rho_g}$ appears valid. Although this differs from the relationship used by Thom (*2*), it should be noted that Thom's correlation was applied only to water in the pressure range well above 200 lb./sq.in.abs. (liquid-to-vapor density ratio less than 125).

The pressure drop Equations (1), (2), and (3) may be combined to yield the following:

$$\Delta P = \frac{G^2}{144\rho_l g_c}\left[R_1 + f_{TP}\left(\frac{L}{D}\right)(R_1 + 2)\right] + R_2\left(\frac{g}{g_c}\right)\frac{\rho_l L}{144} \quad (4)$$

When $V = \sqrt{\rho_l/\rho_g}$ is assumed

$$R_1 = \{1 + x_E\,[\sqrt{(\rho_l/\rho_g)} - 1]\}^2 - 1 \quad (4a)$$

$$R_2 = \frac{\sqrt{(\rho_g/\rho_l)} - 1}{\sqrt{(\rho_l/\rho_g)} - 1} + \frac{\sqrt{(\rho_l/\rho_g)} - \sqrt{\rho_g/\rho_l}}{x_E\,[\sqrt{(\rho_l/\rho_g)} - 1]^2}\ln\{1 + x_E\,[\sqrt{(\rho_l/\rho_g)} - 1]\} \quad (4b)$$

The two-phase friction factor, f_{TP}, was correlated empirically in reference *1*. Since the heat flux was not necessarily uniform and the two-phase friction factor not necessarily constant (as assumed in the integrations), the experimental values of f_{TP} are effective-mean values. The data (*1*) and (*5*) were correlated as follows:

$$f_{TP} = 0.020\left[\frac{DG}{\mu_g}\left(\frac{x_E}{2}\right)\right]^{-0.2}\left\{1 + 0.027\left[\frac{DG}{\mu_l}\left(1 - \frac{x_E}{2}\right)\right]^{0.5}\right\} \quad (5)$$

COMPARISON WITH ALKALI-METAL DATA

After the development of this correlation (*1*), further sodium boiling, pressure-drop data were obtained. Some of these data are shown in Figure 1, where the boiler pressure drop is plotted vs. boiling-fluid mass velocity for various heating rates at a boiler exit temperature of about 1,740°F. ($\rho_l/\rho_g \approx 1{,}500$). Where the experimental conditions are indicated as approximate, the reported values were used in calculating the pressure drop. The calculated curves were obtained by fairing a curve through these calculated data. The experimental data are within 13% of the calculated pressure drop.

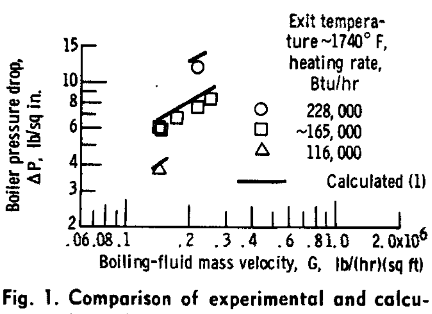

Fig. 1. Comparison of experimental and calculated data for boiling sodium; unpublished NASA experimental data.

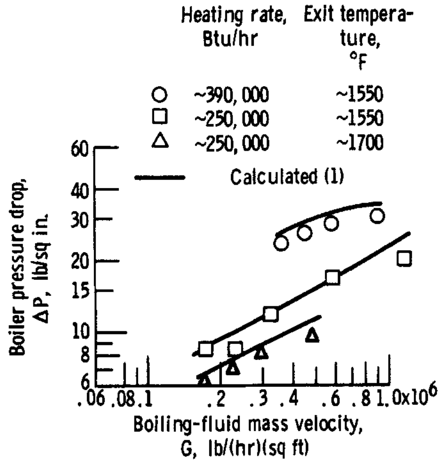

Fig. 2. Comparison of experimental and calculated data for boiling potassium; experimental data of Peterson (6).

As a further test of this correlation, a comparison is made in Figure 2 with the data of Peterson (*6*) for potassium boiling at about 1,550° and 1,700°F. The corresponding ρ_l/ρ_g are approximately 650 and 450 respectively. Boiler pressure drop is plotted vs. boiling-fluid mass velocity (Figure 2) with exit temperature and heating rate indicated as parameters. The experimental data are within 20% of calculated pressure drop.

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NOTATION

- D = inside diameter of boiler tube, ft.
- f_{TP} = two-phase friction factor, dimensionless
- g = acceleration due to gravity, (ft.)/(hr.)^2
- g_c = conversion factor, (lb.$_m$) (ft.)/(lb.$_f$) (hr.)^2
- G = mass velocity of boiling fluid, lb.$_m$/(hr.) (sq.ft.)
- L = length of boiling section, ft.
- ΔP = boiler pressure drop, lb.$_f$/sq.in.
- ΔP_F = frictional pressure drop, lb.$_f$/sq.in.
- ΔP_G = gravitational pressure drop, lb.$_f$/sq.in.
- ΔP_I = inertial (acceleration) pressure drop, lb.$_f$/sq.in.
- R_1 = slip-flow parameter defined by Equation (4*a*), dimensionless
- R_2 = gravitational pressure drop multiplier defined by Equation (4*b*), dimensionless
- V = ratio of mean vapor velocity to mean liquid velocity, dimensionless
- x_E = vapor quality at boiler exit, dimensionless
- ρ_g = vapor density, lb.$_m$/cu.ft.
- ρ_l = liquid density, lb.$_m$/cu.ft.
- μ_g = vapor viscosity, lb.$_m$/(ft.) (hr.)
- μ_l = liquid viscosity, lb.$_m$/(ft.) (hr.)

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